

**The Massachusetts
Comprehensive Assessment
System**

(MCAS)

Release of

2002

Test Items

Mathematics

Grade 8

Mathematics, Grade 8



2002 Massachusetts Comprehensive Assessment System Grade 8 Mathematics Reference Sheet

Use the information and ruler below as needed to answer questions in this test.

PERIMETER FORMULAS

square..... $P = 4s$

rectangle..... $P = 2b + 2h$

triangle..... $P = a + b + c$

CIRCLE FORMULAS

circle..... $C = 2\pi r$

OR

$$C = \pi d$$

$$A = \pi r^2$$

CONVERSIONS

$$1 \text{ mile} = 5280 \text{ feet}$$

$$1 \text{ square mile} = 640 \text{ acres}$$

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

AREA FORMULAS

square..... $A = s^2$

rectangle..... $A = bh$

OR

$$A = lw$$

triangle..... $A = \frac{1}{2}bh$

circle..... $A = \pi r^2$

trapezoid..... $A = \frac{1}{2}h(b_1 + b_2)$

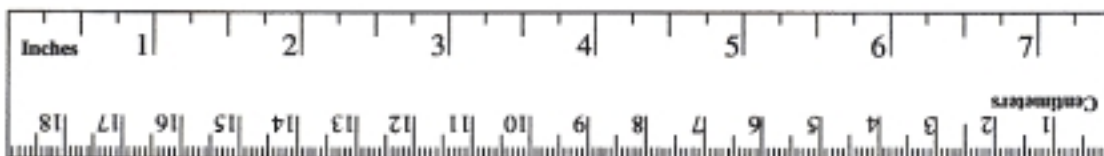
VOLUME FORMULAS

rectangular prism..... $V = Bh$
(B = area of base)

cone..... $V = \frac{1}{3}\pi r^2 h$

cylinder..... $V = \pi r^2 h$

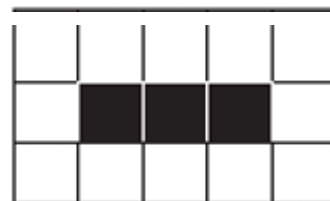
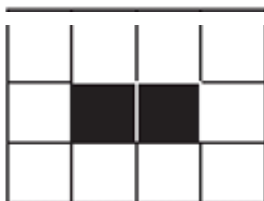
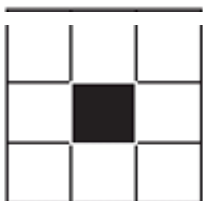
cube..... $V = s^3$
(s = length of an edge)



Session 1, Open Response Question #9 



A worker placed white tiles around black tiles in the pattern shown in the three figures below.



- Based on this pattern, how many white tiles would be needed for 4 black tiles?
- Based on this pattern, how many white tiles would be needed for 50 black tiles?
- Make a scatterplot of the first five figures in this pattern showing the relationship between the number of white tiles and the number of black tiles. Be sure to label the axes.
- Based on this pattern, explain how you could find the number of white tiles needed for any number, n , of black tiles. Show or explain your work.

Reporting Category for item 9: *Patterns, Relations, and Algebra*

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Question 9 Scoring Guide

Score	Description
4	The response shows a comprehensive understanding of how to extend, represent and generalize a pattern with graphs and symbolic expressions.
3	The response shows a general understanding of how to extend, represent and generalize a pattern with graphs and symbolic expressions.
2	The response shows a basic understanding of patterns.
1	The response shows a minimal understanding of patterns.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Score Point 4

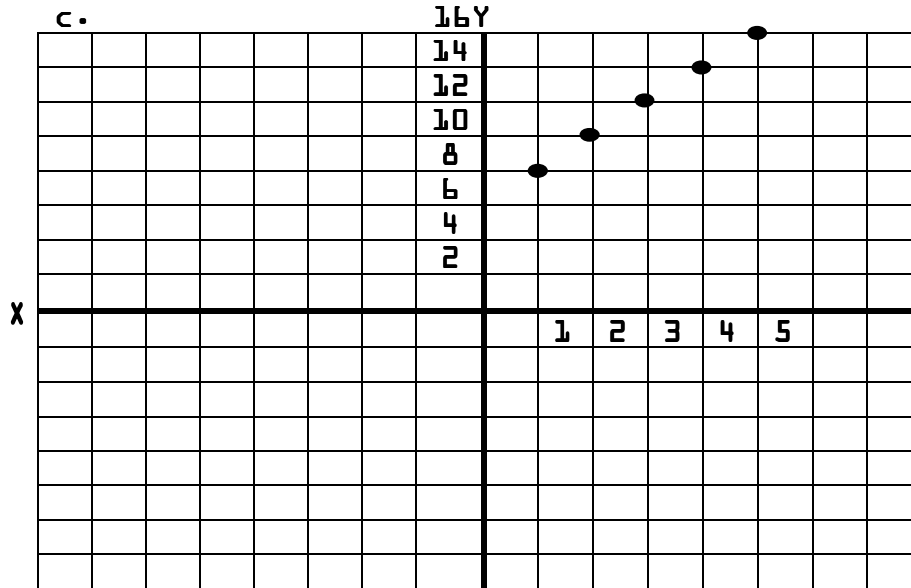
- For a pattern with 4 black tiles you would need 14 white tiles
- For a pattern with 50 black tiles you would need 106 white tiles.
- *See next page.**
- To find the number of white tiles for any number you could use the formula below.

$$\begin{array}{ccccc} & & 2n & + & 6 \\ & \swarrow & | & & \searrow \\ 2(1)+6 & & 2(2)+6 & & 2(3)+6 \\ 8 & & 10 & & 12 \end{array}$$

Mathematics, Grade 8

Score Point #4 continued

x = black tiles
y = white tiles



Score Point 3

- A. 14 white tiles would be needed for four black tiles.
- B. 106 white tiles would be needed to make 50 black tiles.



D. $n = 2b + b$ $B = \text{black}$, $n = \text{white}$

Mathematics, Grade 8

Score Point #3D continued

It took a lot of thought to find this formula. I looked at the white and black tiles closely for quite a while. In all the patterns, there was 12 times the number of black tiles, plus 6. This formula works for every pattern of black and white tiles.

Score Point 2

A= In my opinion, 6 white tiles would be needed for 4 black tiles.

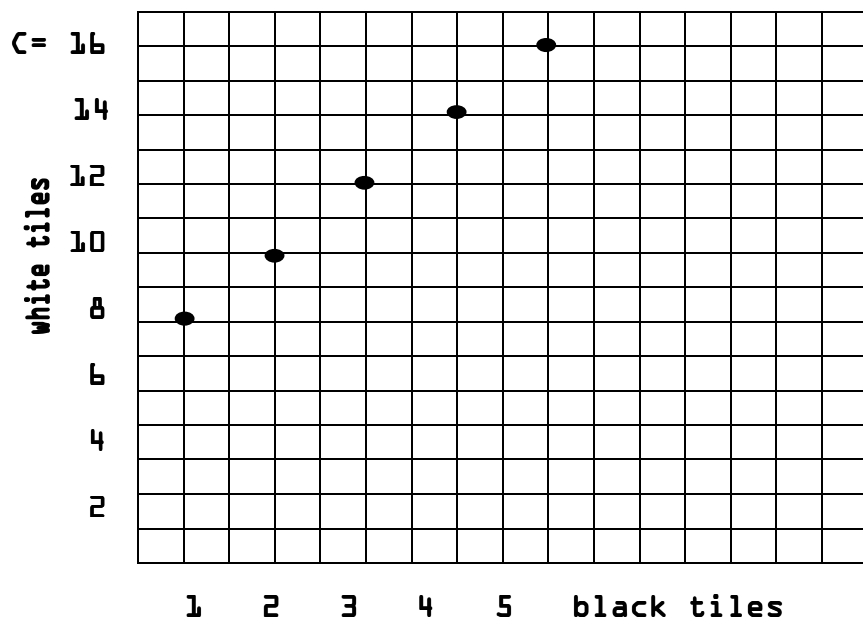
B= There would have to be 100 white blocks.

D= In order to find the letter n , you have to add the number of black tiles to 7. And everytime you add another black block add 1 more to 7. Then you have to add 2 to the 7.

$$\begin{aligned} D= W &= B + 7 + 2 \\ W &= 3 + 7 + 2 \\ 12 &= 3 + 7 + 2 \end{aligned}$$

$$\begin{aligned} \text{also} \\ D= W &= B + 8 + 2 \\ W &= 4 + 8 + 2 \\ 14 &= 4 + 8 + 2 \end{aligned}$$

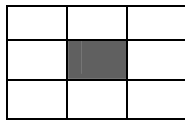
Everytime you add another black block you have to add another to the 7.



Score Point 1

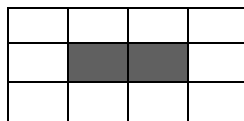
- a. 12 white tiles would be needed for four black tiles.
- b. 150 white tiles would be needed for 50 black tiles.

c. I.



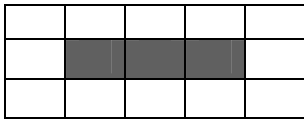
8 white tiles
1 black tile

II



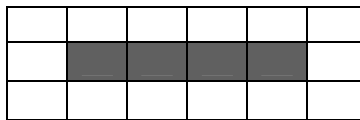
10 white tiles
12 black tiles

III.



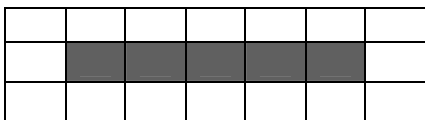
12 w. tiles
3 b. tiles

IV.



14 w. tiles
4 b. tiles

IIV.

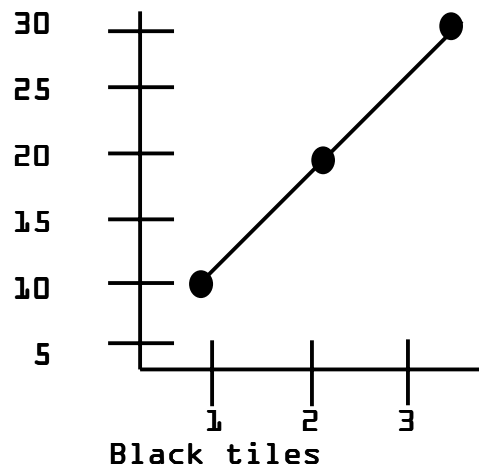


16 w. tiles
5 b. tiles

- d. I could find out the # of tiles needed for every 1 black tile easily. For every one tile there are two other white tiles. For the just one you need to surround it with tiles. Then everytime you add a black one you add two white ones.

Score Point 0

- A. 20 white tiles would be needed for 4 black tiles.
- B. 230 white tiles would be needed for 50 black tiles.
- C. White tiles

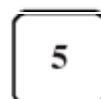
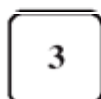
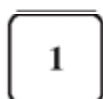


- D. I added all the tiles up or counted all the white squares surrounding the 4 black ones and added 50 to it than x's it by 4. the 4 stands for the 4 black squares.

Session 1, Open Response Question #22 



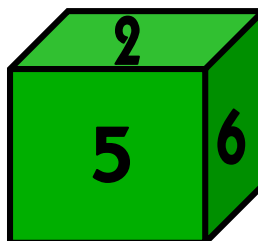
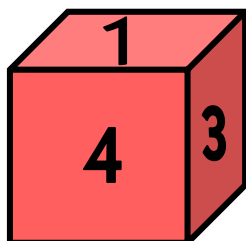
Lionel and Tracy are playing a game using two six-sided number cubes. The faces of each cube are numbered as shown below.



Lionel has a red cube and Tracy has a green cube. To play the game they both roll their cubes at the same time.

- The numbers that show face up when the cubes stop rolling are used to make a fraction.
- The number on the red cube is used for the numerator and the number on the green cube is used for the denominator.

For example, the results shown below would make the fraction $\frac{1}{2}$.



- Lionel wins 1 point if the fraction formed has a value less than one.
- Tracy wins 1 point if the fraction has a value greater than one.
- No one gets a point if the fraction is equal to one.

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Question #22 continued

- a. Make a list or a table in your Student Answer Booklet of all of the fractions possible from rolling 1 red and 1 green cube. How many total different fractions are there?
- b. If Lionel (red cube) rolls a 3, what is the probability that Tracy (green cube) wins 1 point? Show your work or explain how you obtained your answer.
- c. Using your table, what is the probability of each player winning a point on a given turn? Do you think this game is fair to both players? Show your work or explain how you obtained your answer.

Reporting Category for item 22: *Data Analysis, Statistics, and Probability*

Mathematics, Grade 8

Question 22 Scoring Guide

Score	Description
4	The response shows a comprehensive understanding of how to find the total number of outcomes for a situation and calculate the probability of simple compound events.
3	The response shows a general understanding of how to find the total number of outcomes for a situation and calculate the probability of simple compound events.
2	The response shows a basic understanding of probability.
1	The response shows a minimal understanding of probability.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Score Point 4

$\frac{1}{1}$	$\frac{1}{2}$ L	$\frac{1}{3}$ L	$\frac{1}{4}$ L	$\frac{1}{5}$ L	$\frac{1}{6}$ L
$\frac{2}{1}$ T	$\frac{2}{2}$	$\frac{2}{3}$ L	$\frac{2}{4}$ L	$\frac{2}{5}$ L	$\frac{2}{6}$ L
$\frac{3}{1}$ T	$\frac{3}{2}$ T	$\frac{3}{3}$	$\frac{3}{4}$ L	$\frac{3}{5}$ L	$\frac{3}{6}$ L
$\frac{4}{1}$ T	$\frac{4}{2}$ T	$\frac{4}{3}$ T	$\frac{4}{4}$	$\frac{4}{5}$ L	$\frac{4}{6}$ L
$\frac{5}{1}$ T	$\frac{5}{2}$ T	$\frac{5}{3}$ T	$\frac{5}{4}$ T	$\frac{5}{5}$	$\frac{5}{6}$ L
$\frac{6}{1}$ T	$\frac{6}{2}$ T	$\frac{6}{3}$ T	$\frac{6}{4}$ T	$\frac{6}{5}$ T	$\frac{6}{6}$

36 different
Fractions

tie = blank
T = Tracy wins
L = Lionel wins

Possibilities $\frac{3}{1}$ $\frac{3}{2}$ $\frac{3}{3}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{3}{6}$

Winner T T tie L L L

Mathematics, Grade 8

Score Point #4 continued

$$\frac{2}{6} = \left(\frac{1}{3} \right)$$

$$\frac{15}{36} \text{ Tracy Wins} = \frac{5}{12} \quad \text{It is a fair game.}$$

$$\frac{15}{36} \text{ Lionel Wins} = \frac{5}{12} \quad \text{The probabilities are equal.}$$

$$\frac{6}{36} = \frac{1}{6}$$

Score Point 3

- A. for the number 2 there are 6 possible fractions. That is the same as for all the numbers on the dice (there are six). That means there are 36 possible fractions.
- B. Possible combinations -

$\frac{3}{1}$	$\frac{3}{2}$	$\frac{3}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{3}{6}$
>1	>1	=1	<1	<1	<1

$$\frac{2}{6} = \frac{1}{3}$$

Tracy has a $\frac{1}{3}$ chance to get one point if Lionel rolls a 3.

- C. 15 possible fractions out of 36 are greater than 1. That means $\frac{5}{12}$ are >1. I used my table to figure that out.

This game is fair because both people can get $\frac{5}{12}$ chances to get a point. It would not be fair if I counted as a point.

Mathematics, Grade 8

Score Point 2

A.	1,1	2,4	4,1	5,4
	1,2	2,5	4,2	5,5
	1,3	2,6	4,3	5,6
	1,4	3,1	4,4	6,1
	1,5	3,2	4,5	6,2
	1,6	3,3	4,6	6,3
	2,1	3,4	5,1	6,4
	2,2	3,5	5,2	6,5
	2,3	3,6	5,3	6,6

} There are a total of 36 combinations.

- B. If Lionel rolls a (red) 3 there is a 50/50 chance that Tracy will win the point. This is true because it is half way between getting a six or one.

I think this game is fair to both players, although Lionel is more likely to win. He has more chances of winning because there are a few more combinations in his favor.

Score Point 1

If some are rolling different colors, then there would be $\frac{1}{6}$ of all different fractions. If Lionel rolls a 3, Tracy would roll she would probably get 1 out of 3 chances so it is very likely that she will get a point because she has several chances. The probability of each player winning a point on a given turn is 1 out of 6 chances. I think it is not fair to both players because even though you have a couple of chances you still have 1 out of 6. Since there is two people playing then it would be 1 out of 6 means that only one person can win because, they can't tie.

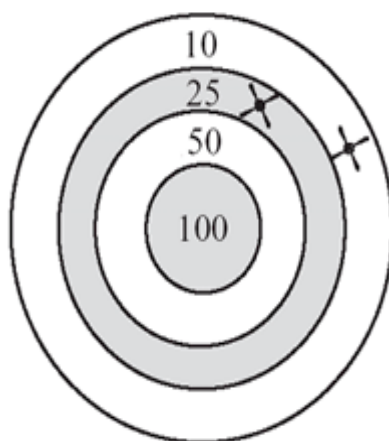
Score Point 0

- (A) 12345
13452
14523
15234
15432 (B)
14325
13254
12543
15324
- (C) Yes I do why shoulnt they the end bye bye 2.

Session 2, Open Response Question #28



Esther shot two arrows at a target.
The picture below shows where the arrows landed.



Esther calculated her score by adding the number of points for each ring in which an arrow landed. For the two arrows above, her score was 35 points ($25 + 10$).

- In your Student Answer Booklet, make a list of all the possible scores Esther could have gotten by shooting two arrows that hit the target.
- Is it possible for Esther to score a total of 235 points using **only** 5 arrows? Show your work or explain your answer.
- What is the **fewest** number of arrows required for Esther to score a total of 240 points? Show your work or explain your answer.

Reporting Category for item 28: *Number Sense and Operations*

Mathematics, Grade 8

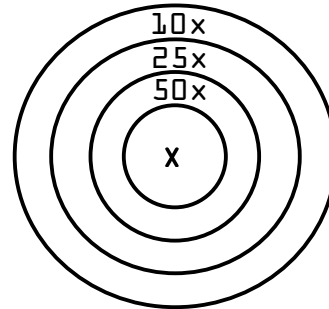
Question 28 Scoring Guide

Score	Description
4	The response shows a comprehensive understanding of how to select and use appropriate operations to solve problems.
3	The response shows a general understanding of how to select and use appropriate operations to solve problems.
2	The response shows a basic understanding of operations.
1	The response shows a minimal understanding of operations.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

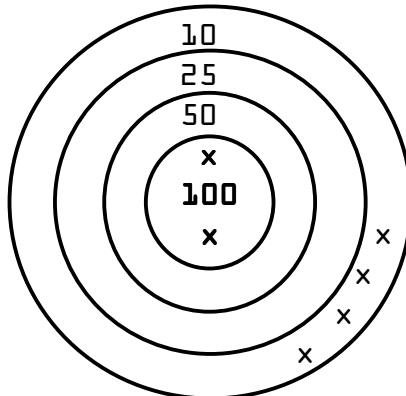
Score Point 4

a. 200 50
 150 35
 125 20
 110
 100
 75
 60

b. Yes



c. b



Score Point 3

A

arrow #1	10	25	50	100	10	25	50	100	10	25	50	100
arrow #2	10	25	50	100	25	50	100	10	50	100	10	25
arrow #1	10	25	50	100								
arrow #2	100	10	25	50								

There are 16 possible combinations Ester could have shot. 20, 50, 100, 200, 125, 75, 150, 110, 60, 35 are the sums that both arrows together could have equaled.

B

No, Ester could not score a total of 235 points using 5 arrows. Using 5 arrows there are no combinations or numbers that Ester could shoot that would equal 235. For example using 5 arrows ester could have a total of 220, or 250 but there are no number combinations that would equal 235.

C

The fewest number of arrows required for ester to get 240 points is 6 arrows. Ester could get 200 points with his first two arrows (100 points each) and then 10 points with each of the remaining four arrows. This equals 240 points.

Mathematics, Grade 8

Score Point 2

a)

100	100	100	100	100	100
100	50	50	25	25	10
50	50	50	25	25	10
50	25	10	25	10	10

200
150
125
110
100
75
60
50
35
20

b)

100
100
100

100 min
25 min
+10
135

Yes
ester could
have got a
100, 25, 10
and missed
the other 2.

c)

100	1
10	2
10	3
10	4
10	5

The fewest would be 4
arrows to get a total of
of 14 points.

Score Point 1

A ↓

10 + 25 = 35
10 + 50 = 60
10 + 100 = 110
25 + 50 = 75
25 + 100 = 125
50 + 50 = 100
50 + 100 = 150
10 + 10 = 20
25 + 25 = 50

Mathematics, Grade 8

Score Point #1 continued

- b. No, because if she uses 5 it will go over 235 but if she does less than 5 it will be under 235.
- c. the fewest number of arrows would be 5 because you need $35 + 50 + 60 + 75 + 20 = 240$.

Score Point 0

a. She could had got

$$\begin{array}{r} 10 \\ 25 \\ +50 \\ \hline 100 \\ 150 \\ +35 \\ \hline 185 \\ \times 2 \\ \hline 370 \end{array}$$

- b. No because she would have had 370.
- c. It would be only 1 arrow.

Session 2, Open Response Question #29



Molly formed three polygons—a triangle, a rectangle, and a pentagon—with string. She calculated the sum of the measures of the interior angles for each polygon and entered her data in the chart shown below.

Type of Polygons	Number of Sides	Sum of the Measures of the Interior Angles
Triangle	3	180°
Rectangle	4	360°
Pentagon	5	540°
Hexagon	6	?
Octagon	8	?
Unnamed Polygon	?	2340°
n -sided Polygon	n	?

- What is the sum of the measures of the interior angles of a hexagon?
- What is the sum of the measures of the interior angles of an octagon?
- How many sides does an unnamed polygon have if the sum of the measures of the interior angles is 2340°?
- Explain how you would find the sum of the measures of the interior angles of an n -sided polygon.

Reporting Category for item 29: **Geometry**

Question 29 Scoring Guide

Score	Description
4	The response shows a comprehensive understanding of the relationship between the number of sides and the sums of angle measures of polygons.
3	The response shows a general understanding of the relationship between the number of sides and the sums of angle measures of polygons.
2	The response shows a basic understanding of the relationship between the number of sides and the sums of angle measures of polygons.
1	The response shows a minimal understanding of the relationship between the number of sides and the sums of angle measures of polygons.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Score Point 4

- (a) The sum of the measures of the interior angles of a hexagon are 720° .
- (b) The sum of the measures of the interior of an octagon are 1080° .
- (c) There are 15 sides in the un-named polygon whose interior angles add up to 2340° .
- (d) To find the measure of an n-sided polygon, I used the formula $[(n-2)2] 90$
2 is subtracted from n and the new number is multiplied by 90 to get the sum of the measures of all the interior angles.
To figure out the numbers of sides, divide the measure of all the interior angles by n.

Score Point 3

- A. The interior angle is 720° Due to you add 180° each time you change shapes.
- B. The sum of a octagon is 1080° .
- C. It MUST Have 15 sides to add up to 2340° .
- D. I would add 180° for each side on the pollygon.

Score Point 2

A

Inteor angles of a hexagon is 720° I found this by adding 180° to 540° because of the hexagon having 6 sides.

B

Interior angles of an octagon = 1080° . I found this by adding 360° to 720° . I did this because the sides of an octagon increased by 2.(8).

C

The sides of the unnamed polygon are 13 sides. The way I figured this out was by first multiplying 180° by 14. This gave me 2520° . I was close, but that answer was not right. So, I then multiplied 180 by 13. This was the correct number I needed.

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Score Point #2 continued

D

The way I thought you could find the measure of an n -sided polygon was knowing that as many times the sides measure, you add, double, or even triple 180° and add it on to the number of angles for the shape of before. For example, a triangle had 3 sides so you start of f with 180° . Then a rectangle had four sides so you add 180 to that giving you 360° : Then you could go from a rectangle to a hexagon which has 3 more than a rectangle (6). By doing this you then multiply 6 by 180 giving you 720° . In conclusion, by using this method, you will be able to find the measures of the interior angles.

Score Point 1

- (a) Hexagon = 720° , (b) Octagon = 900° ;
(c) 10 sides for the unnamed polygon;
(d) You would 360 to the unnamed polygon to get the number of the sum of the measures of the interior angles.

Score Point 0

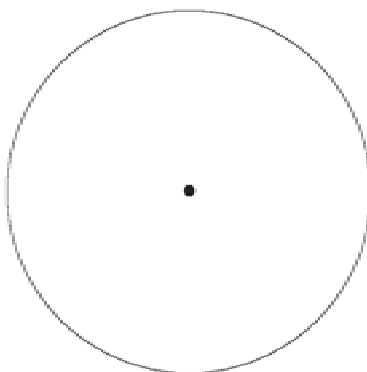
- (A) The sum of the measures of the Interior angle for a hexagon is 1080° because you would do 6 (for the 6 sides) $\times 180^\circ$ which is equal to 1080° .
(B) There is 1440° in a octagon because 8 (for sides) $\times 180^\circ$ equal 1440° .
(C) 13 sides in a unnamed polygon because it is 2340° in it so you do $2340 \div 180 = 13$.
(D) If you # of sides was 10 you would do $10 \times 180 = 1800^\circ$.

Session 2, Open Response Question #39



Use the ruler included in your reference sheet to answer question 39.

The figure shown above represents the base of a cylindrical tank. The tank has a height of 16 centimeters (1 milliliter = 1 cubic centimeter).



- What is the radius of the base, in centimeters?
- What is the volume of the cylinder in milliliters? Show your work.
- If both the radius and the height of the cylinder were doubled, what would be the volume of the cylinder in milliliters? Show your work.
- Based on your answers to parts b and c, what is the ratio of the volume of the smaller tank to the volume of the larger tank? Show your work.

Reporting Category for item 39: Measurement

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Question 39 Scoring Guide

Score	Description
4	The response demonstrates a comprehensive understanding of how a change in one variable results in a change in another variable by correctly answering the four parts of the question.
3	The response demonstrates a general understanding of how a change in one variable results in a change in another variable by correctly answering three parts of the question.
2	The response demonstrates a partial understanding of how a change in one variable results in a change in another variable by correctly answering two parts of the question.
1	The response demonstrates a minimal understanding of how a change in one variable results in a change in another variable by correctly answering one part of the question.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No Response.

Score Point 4

a. $r = 5\text{cm}$

b. $3.14 \cdot 5^2 \cdot 16$
 $3.14 \cdot 25 \cdot 16$
 $78.5 \cdot 16$

1,256 milliliters

c. $3.14 \cdot 10^2 \cdot 32$
 $3.14 \cdot 100 \cdot 32$
 $3.14 \cdot 32$
10,048 milliliters

d. $\frac{1256}{10048} \rightarrow \frac{628}{5024} \rightarrow \frac{314}{2512} \rightarrow \frac{157}{1256}$

ratio = $\frac{157}{1256}$

Mathematics, Grade 8

Score Point 3

A. the radius is 5 centimeters

B. $v = \pi r^2 h$

$$v = \pi^5 16$$

$$v = \pi \times 25 \times 16$$

$$v = 79 \times 16$$

$$v = 1264 \text{ centimeters}^2$$

$$1264 \times 100$$

$$126,400 \text{ milliliters}$$

C. $v = \pi \times 100 \times 32$

$$v = \pi 3200$$

$$v = 10,053 \text{ cent.}$$

$$10053 \times 100 =$$

$$1,005,300 \text{ milliliters}$$

D. $1,005,300 > 126,400$

$$\frac{1005300}{126400} \approx 8 \text{ times}$$

$$L = \pi 2r^2 \times 16 = 2 \sqrt{\quad}$$

$$\text{small} = S \quad \text{large} = L$$

$$S8 = L$$

Score Point 2

Problem 39

(a) 100 cm^2

(b) $v = 502400 \text{ milliliters}$

$$3.14 \times 100\text{cm}^2 \times 16\text{cm}$$

$$31400 \times 16 = 502400$$

$$r = 200$$

$$H = 32$$

$$v = 4019200$$

$$3.14 \times 200^2 \times 32$$

(c) ratio 3.21

Score Point 1

- A. The Radius of the base is 15^{cm}. I found this by measuring from the center of the circle to the wall which gave the Radius.
- B. The volume of the cylinder is 418.66664^{m^l}. This is how I got that answer:
$$3.14 \div 3 \cdot 25 \cdot 16 = 16^{\text{m}^{\text{l}}}.$$
- C. If both the Radius and height were doubled the new volume would be 3349.3331^{m^l}. This is how I got my answer:
$$3.14 \div 3 \cdot 100 \cdot 32 = 3349.3331$$
- D. I have no Idea how to do a ratio. I have a vague Idea what one is, but I don't have a chance of doing this problem correctly if I don't know what a Ratio is or how to figure one out.

Score Point 0

The radius of the base, in centimeters is 9.9. The volume of the cylinder in milliliters is 10. I rounded 9.9 to 10. If both the radius and the height of the cylinder were doubled, the volume of the cylinder in milliliters would be 42. I just added 10 + 16 + 16. I added 16 twice because it said "the height of the cylinder were doubled. Add 16 + 16 which equals 32 to 10 because that is the volume of the cylinder in milliliters and you get 42. The ratio to the smaller tank to the volume of the larger tank would be 21. I divided 42 by 2 to get that because the smaller tank would be smaller so just divide by the larger tank to get the answer to the smaller tank and you get 21.

**The Massachusetts
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Release of

2001

Test Items

Mathematics

Grade 8

Mathematics, Grade 8

2000-2001 Massachusetts Comprehensive Assessment System Grade 8 Mathematics Reference Sheet

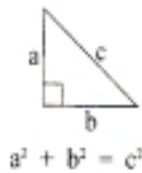
Use the information and ruler below as needed to answer questions in this test.

PERIMETER FORMULAS

square: $P = 4s$

rectangle: $P = 2b + 2h$

triangle: $P = a + b + c$



CIRCLE FORMULAS

circle: $C = 2\pi r$

OR

$C = \pi d$

$A = \pi r^2$

CONVERSIONS

1 mile = 5280 feet

1 square mile = 640 acres

AREA FORMULAS

square: $A = s^2$

rectangle: $A = bh$

triangle: $A = \frac{1}{2}bh$

circle: $A = \pi r^2$

trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

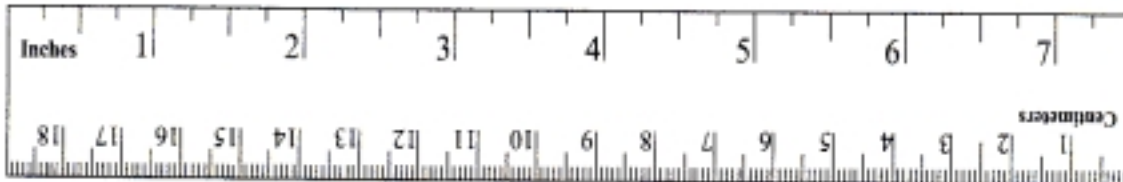
VOLUME FORMULAS

rectangular prism: $V = Bh$
(B = area of base)

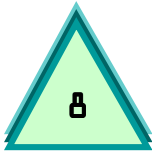
cone: $V = \frac{\pi}{3}r^2h$

cylinder: $V = \pi r^2h$

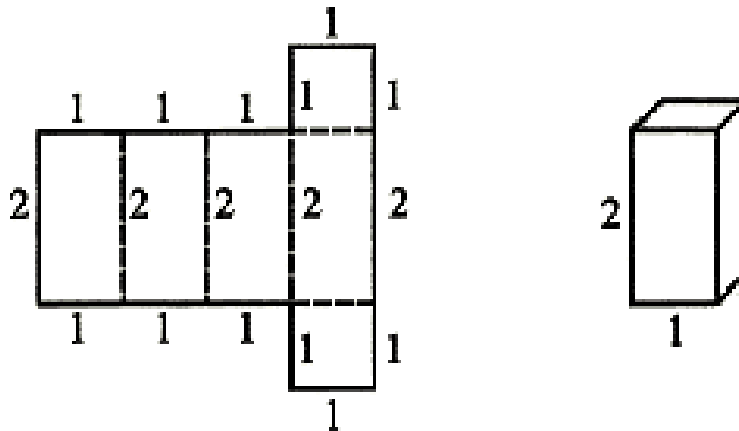
cube: $V = e^3$
(e = length of an edge)



Session 1, Open Response Question #8



The pattern shown below is for a square prism. The lengths of the line segments in the pattern were chosen so that the pattern could be folded along the dotted lines into the prism shown.



- Make a sketch of a pattern for a triangular prism. Label each line segment with a length that will make it possible to fold the pattern into the triangular prism.
- Make a sketch of a pattern for a cylinder. Label each line segment and diameter in your pattern with a length that will make it possible to create the cylinder from the pattern.

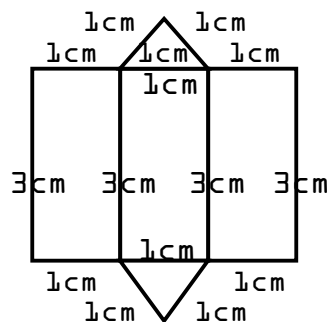
Reporting Category for item 8: **Geometry**

Question 8 Scoring Guide

Score	Description
4	Student shows comprehensive sense of spatial relationships by making accurate sketches of patterns for two three-dimensional geometric figures and labeling the lengths of edges appropriately.
3	Student shows good sense of spatial relationships by making accurate sketches of patterns for two three-dimensional geometric figures and labeling the lengths of edges. One or two significant measures may be omitted.
2	Student shows partial sense of spatial relationships by inconsistently making sketches of patterns for two three-dimensional geometric figures or inconsistently labeling the lengths of edges.
1	Student shows limited sense of spatial relationships by making major errors in sketches and labeling.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

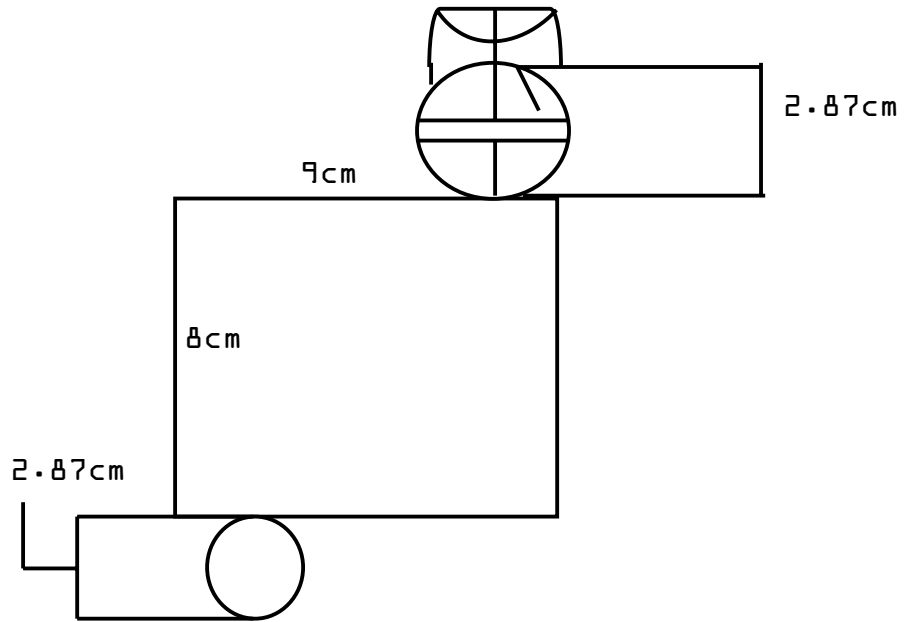
A.



Mathematics, Grade 8

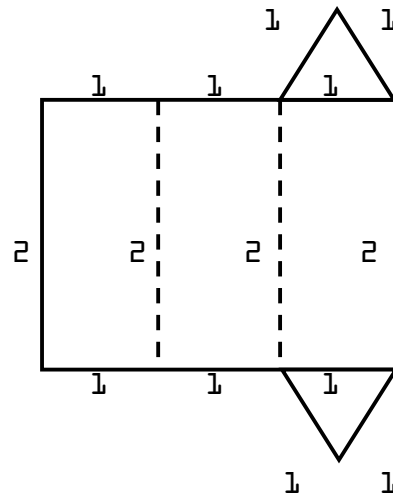
Score Point #4 continued

B.



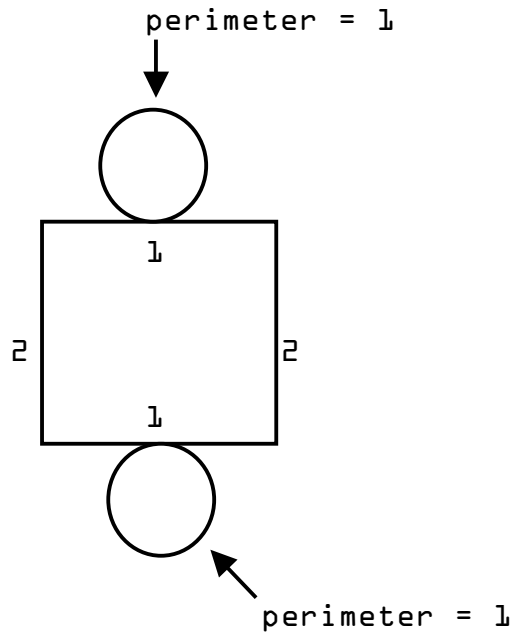
Score Point 3

a.



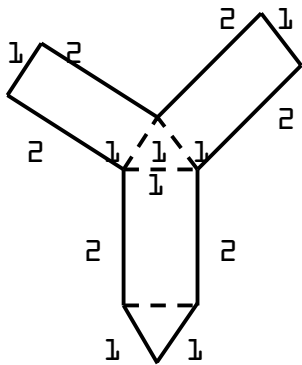
Score Point #3 continued

b.

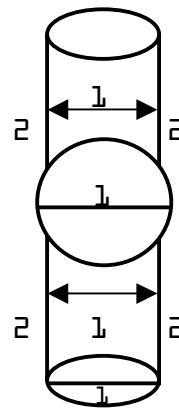


Score Point 2

a.

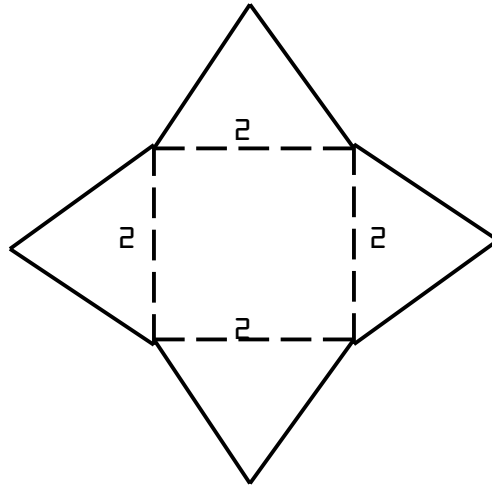


b.

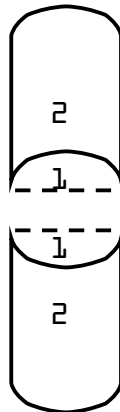


Score Point 1

a.

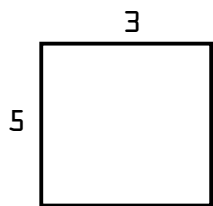
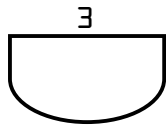
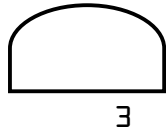
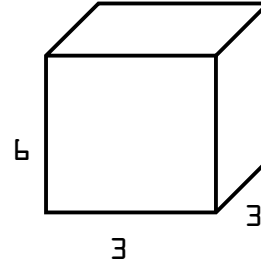
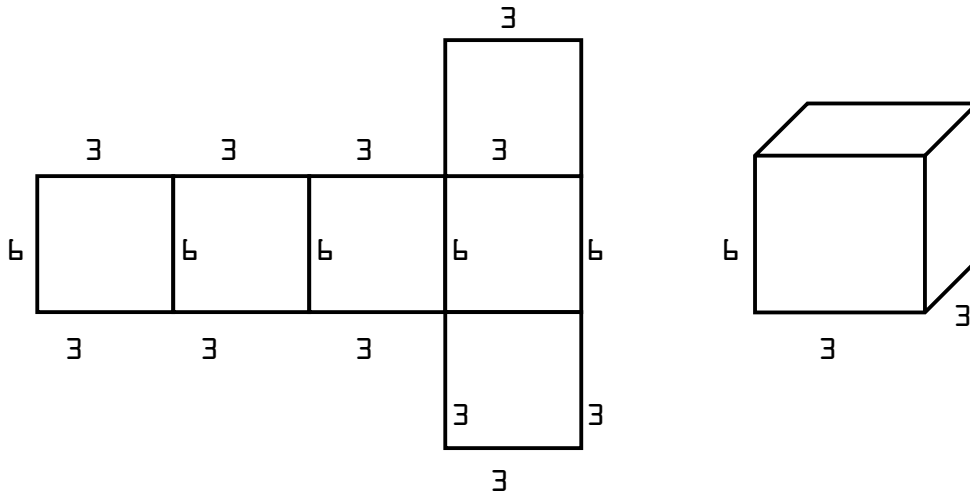


b.

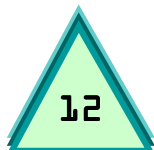


Score Point 0

a)



Session 1, Open Response Question #12



An eighth-grade class will perform the first **four** acts in the annual talent show. Every student is in exactly one of the four acts. The order in which the acts will be presented is to be decided by a drawing so that each act has an equal chance of being drawn.

- a. Chantal is a member of the eighth-grade class. What is the probability that her act will be presented first?

- b. Chantal's act was chosen to be presented first. Make a tree diagram, chart, or list showing all the possible orders in which the **other three acts** could be presented. Use the letters A, B, and C to represent these three acts.

- c. Rory, Jesse, and Chantal are all members of the eighth-grade class who will each perform an act. What is the probability that Rory's act will immediately follow Jesse's? Explain how you found your answer.

Reporting Category for item 12: Analysis, Statistics, and Probability

Question 12 Scoring Guide

Score	Description
4	The response demonstrates comprehensive understanding of the concepts of probability and basic combinatorics by accurately describing outcomes and events and determining the probability of those events.
3	The response demonstrates general understanding of the concepts of probability and basic combinatorics by describing outcomes and events and determining the probability of those events.
2	The response demonstrates basic understanding of the concepts of probability and basic combinatorics by describing outcomes and events and/or determining the probability of events.
1	The response demonstrates minimal understanding of the concepts of probability and basic combinatorics by describing outcomes and events and/or determining the probability of events.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

- A. The probability that her act will be presented first is $\frac{1}{4}$ of a 25% chance. The ways the three other acts could be presented are:

- A, B, C
- A, C, B
- C, A, B
- B, A, C
- B, C, A
- C, B, A

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Score Point #4 continued

- C. The possibility that Rory's act will immediately follow Jesse's act is $\frac{1}{3}$. This is because there are six possible placement for the three acts following Chantel. Out of the six possibilities Rory is only right behind Jesse twice. This makes the possibility $\frac{2}{6}$ which can be reduced to $\frac{1}{3}$.

Possible Placements

C, S, R, O

C, J, O, R $\frac{2}{6}$ or $\frac{1}{3}$

C, R, J, O

C, O, J, R

C, O, R, J

C, R, O, J

J = Jesse

O = Other

R = Rory

C = Chantal

Score Point 3

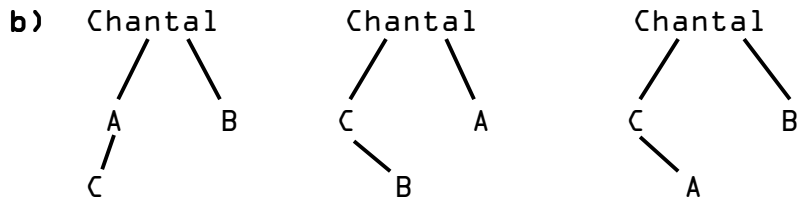
- a) The probability that her act will be presented First is a 1 out of four chance. This is because there is four acts in total, and her act is only one. so, she has a 1 out of four chance, or a $\frac{1}{4}$ chance.
- b)
1. A, B, C
 2. A, C, B
 3. B, A, C
 4. B, C, A
 5. C, A, B
 6. C, B, A
- c) There is a 1 out of 6 chance that Rory's act will immediately follow Jesse's, because there are a total of six orders they can go in and only one out of the six orders would mean Rory goes after Jesse. So, there is a 1 out of 6 chance.

Score Point 2

- A. If there are four acts and she's just one of the 8th graders, the probability that her act will be presented first is $\frac{1}{4}$.
- B. Possible order for next 3 acts
- AB
 - AC
 - BA
 - BC
 - CA
 - CB
- C.

Score Point 1

a) $\frac{1}{4}$ chances



c) 50% chance. If Jesse's goes first then there are only Rory & Chantal to compete next. Which is 50/50.

Score Point 0

Chantal's possibility that her act would get presented first is 1 out of 3. The other groups can have a better chance of their group getting performed that if Chantal's group was not done doing what they had to get ready to perform, that's how the other group could get put in front of Chantal's group. The probability of Rory's group following Jesse's group 1 out of 2, because if Chantal's group is done her group might go before Jesse but this probability is 1 out of 2. Unless Jesse's group is nervous and doesn't want to go right after Rory's group and that's how Chantal's group will go after Rory's group.

Session 2, Open Response Question #23 



An eighth-grade class took a survey and found that the most popular types of music in their school were alternative rock, rap, and classic rock. They took a second survey to find out the students' preference among these three types of music. These are the results for 120 students.

Favorite Types of Music		
Alternative	Rock Rap	Classic Rock
60	40	20

- Make a rough sketch of a circle graph displaying these data. Tell how many degrees should be in each sector of the graph.
- Explain how you find the number of degrees for each sector.

Reporting Category for item 23: *Data Analysis, Statistics, and Probability*

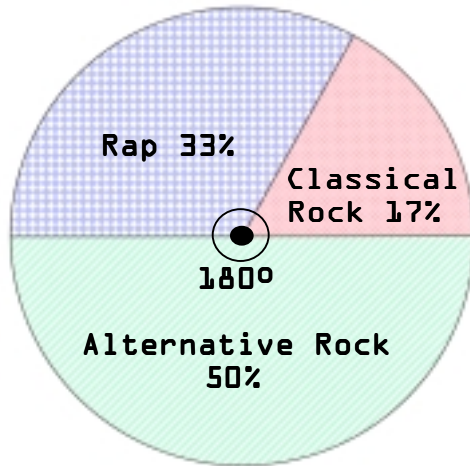
Mathematics, Grade 8

Question 23 Scoring Guide

Score	Description
4	Student makes reasonable sketch of circle graph and specifies correct number of degrees for each sector. Explanation or work is clear and shows correct strategy.
3	Student makes reasonable sketch. Explanation or work shows correct strategy for determining number of degrees. Errors are clearly careless. OR Student makes reasonable sketch and specifies correct number of degrees. Explanation is vague but indicates correct strategy.
2	Student makes reasonable sketch and gives correct percent for each sector, but no degrees. Explanation may or may not be included. OR Student specifies correct number of degrees for each sector. Explanation is incomplete, hard to follow, or missing. Sketch may or may not be included. OR Student's explanation shows understanding of strategy that can be used for determining number of degrees in a sector other than the 180° sector for alternative rock. Sketch may or may not be included. OR Student makes reasonable sketch and gives clear explanation based on fractional parts of circle. Answer has no conceptual errors regarding percents or degrees.
1	Student's sketch of graph shows some understanding of data. OR Response shows a minimal understanding of problem.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

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Score Point 4



$$\frac{40}{120} = \frac{x}{360}$$

$$860 \times 40 \div 120 = 120$$

$$\frac{20}{120} = \frac{x}{360}$$

$$360 \times 20 \div 120 = 60$$

boys who like
alternative rock

how many
degrees?

Cross multiply
 $360 \times 60 \div 120 = 1800$

$$\frac{60}{120} = \frac{x}{360}$$

total number
of boys

degrees
in a circle

$$\frac{40}{120} = \frac{x}{360}$$

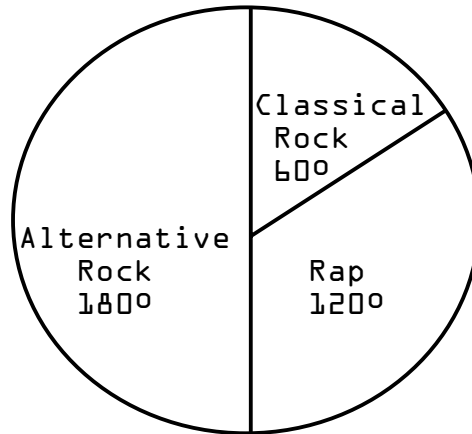
$$360 \times 40 \div 120 = 1200$$

$$\frac{20}{120} = \frac{x}{360}$$

$$360 \times 20 \div 120 = 600$$

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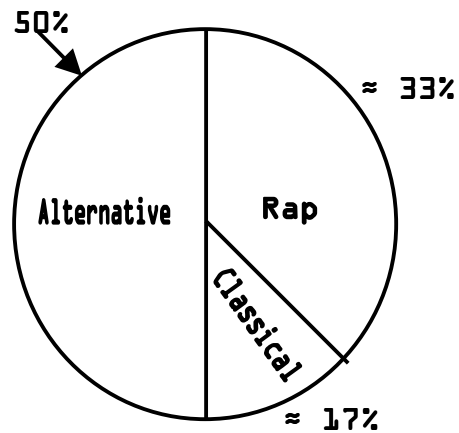
Score Point 3



First, I divided the amount of boys that like Alternative Rock into 120, and saw that it was half of 120. So I then divided that into 360, because that is the amount of degrees in a circle, and saw that there should be 180° for alternative rock. I then went back and did the same thing for classical rock and rap.

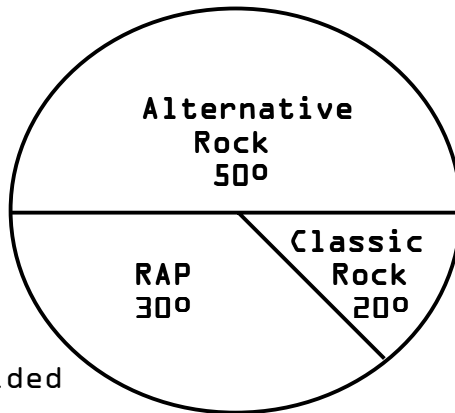
Score Point 2

You take the number of boys and put them into a fraction so say alternative rock you were trying to figure out you would put 60 over 120.



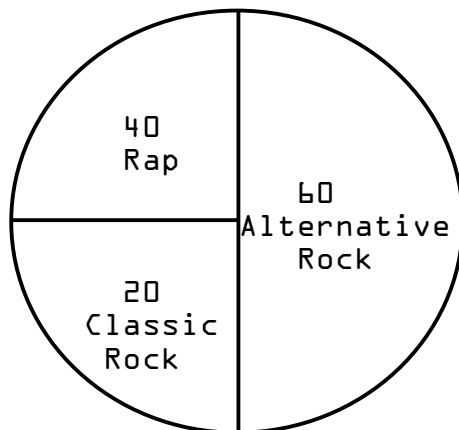
Score Point 1

A.



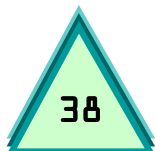
B. I divided

Score Point 0



Alternative rock is the most favorite in the school so it gets the most votes then comes rap then classic rock.

Session 3, Open Response Question #38



Ms. McCarthy's class is making up number puzzles.
These are two of the puzzles.

Manuel's puzzle:

My number is even.
It is a factor of 198 and a multiple of 9.
It is less than 100.
What is my number?

Haan's puzzle:

My number is the product of three different prime numbers.
It is an odd number less than 125.
The sum of its digits is a multiple of 3.
One of its factors is the third prime number.
What is my number?

- a. What is Manuel's number?
- b. What is Haan's number? Explain the strategy you used to find your answer to Haan's puzzle.
- c. Write a number puzzle that
 - has exactly **three** clues,
 - has **one and only one** answer, and
 - includes the following words: **factor** and **prime number**.

Reporting Category for item 38: Number Sense and Operations.

Mathematics, Grade 8

Question 38 Scoring Guide

Score	Description
4	The response demonstrates a thorough understanding of number theory concepts of prime numbers, factors, and multiples and their use in solving problems.
3	The response demonstrates a general understanding of number theory concepts of prime numbers, factors, and multiples and their use in solving problems.
2	The response demonstrates a partial understanding of number theory concepts of prime numbers, factors, and multiples and their use in solving problems.
1	The response demonstrates a minimal understanding of number theory concepts of prime numbers, factors, and multiples and their use in solving problems.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

- a. Manuel's number is 18 because 18 is even, is a factor of 198 ($198 \div 18 = 11$), is a multiple of 9 ($9 \times 2 = 18$), and is less than 100.
- b. Haan's number is 105 because it is a product of 3 different primes ($3 \times 5 \times 7 = 105$), an odd number less than 125, the sum of the digits is a multiple of 3 ($1 + 0 + 5 = 6$, $3 \times 2 = 6$), and one of the digits is the third prime factor. I got the answer by process of elimination and trial and error. I knew that one of the primes couldn't be 2 (that would make it an even number), so the one I tried was $3 \times 5 \times 7$ and it worked.
- c. My number is odd between 0 and 30.
My number is NOT prime.
My number has 7 as a factor.
What is my number?
Answer: 21

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Score Point 3

a. $n = \text{even}$ $n \cdot x = 198$ $\frac{n}{9} = 2$

$$\begin{array}{r} 99 \\ 2 \overline{)198} \\ \underline{180} \\ 18 \end{array}$$

$$\begin{array}{r} 66 \\ 3 \overline{)198} \\ \underline{180} \\ 18 \end{array}$$

$n < 100$ $99 = \text{even}$

$$\begin{array}{r} 99 \\ 4 \overline{)198} \\ \underline{160} \\ 38 \\ \underline{26} \end{array}$$

$56 \checkmark 100$ $56 \checkmark \text{even}$ $56 \times \checkmark 198$ $\frac{56}{9} \not\checkmark 6$

answer: 56

b. $y \cdot y \cdot z = n$ $x, y, z = \text{prime}$ $n = \text{odd}$ $n < 125$

~~$1 \times 5 \times 3 = 15$~~

$\frac{n}{5}$ ~~$2/5$~~ $2+5=7$ ~~$1/5$~~ $1+5=6$ ~~$3/5$~~ $3+5=8$ 45 $4+5=9$

105 $5 \times 7 \times 3 = 105$

$105 \checkmark 150$ $1+0 \checkmark 5 = \frac{6}{8} = 2$ $\frac{105}{5} \checkmark = 21$ $5 \times 7 \checkmark 3 = 105$

$5 \times 7 \times 3 = 105$ **answer: 105**

I used the strategy of multiplying the first 3 odd prime numbers to get 105, then checked it to see if it worked for the rest of the rules.

c. My number is a prime number.

- H is a factor of 121
- H is less than 50

answer: 11

Mathematics, Grade 8

Score Point 2

- a) 18
- b) 15. I figured the third prime number is 5. I then thought of two other prime numbers that were lower than 5. I got 3 and 1.
 $5 \times 3 \times 1 = 15$
- c) My number is not even.
It is a factor of 26.
It is a prime number.
The answer is 13.

Score Point 1

- a) Manuel's number is 22.
- b) Haan's number is 15.
I took the first three prime numbers and found the answer. Guess and check method.
- c) even number
sum of first two prime numbers
has 1 pair of factors that are the same
answer = 4

Score Point 0

- a)
- b)
- c) my number is even it is less than 20
anything multiplied by it is doubled
what is my number
my number is a prime number

Session 3, Open Response Question #39



Some eighth-grade students want to raise at least \$300 for a field trip by selling popcorn and fruit bars. The chart below shows the amount of profit they will make on each sale.

Profit from Sales

Box of popcorn	60¢
Fruit bar	30¢

- If they sell exactly 500 fruit bars, how many boxes of popcorn will they need to sell to make a total of \$300?
- On the grid in your Student Answer Booklet, draw a graph showing the combinations of boxes of popcorn and fruit bars they must sell to make a total of exactly \$300. Let the horizontal axis represent the number of fruit bars. Label that axis to 1,000. Show or describe the calculations you used to find the data points for your graph.
- Based on last year's sales, the students will probably not be able to sell more than 600 fruit bars. **Using your graph**, explain how you can find the number of boxes of popcorn the students must sell to make a total of \$300 if they sell exactly 600 fruit bars. How many boxes of popcorn must they sell?

Reporting Category for item 39: *Patterns, Relations, and Algebra.*

Question 39 Scoring Guide

Score	Description
4	Student demonstrates comprehensive understanding of functional relationships by representing a real-life situation accurately in an appropriate graph, explaining the strategy used, and using the graph to find related values.
3	Student demonstrates general understanding of functional relationships by representing a real-life situation in an appropriate graph, explaining the strategy used, and using the graph to find related values with minor inaccuracies.
2	Student shows basic understanding of functional relationships by representing a real-life situation in an incomplete graph, ineffectively explaining the strategy used, and/or failing to use the graph to find related values.
1	Student demonstrates minimal understanding of the relationship between the variables or of graphing.
0	Response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

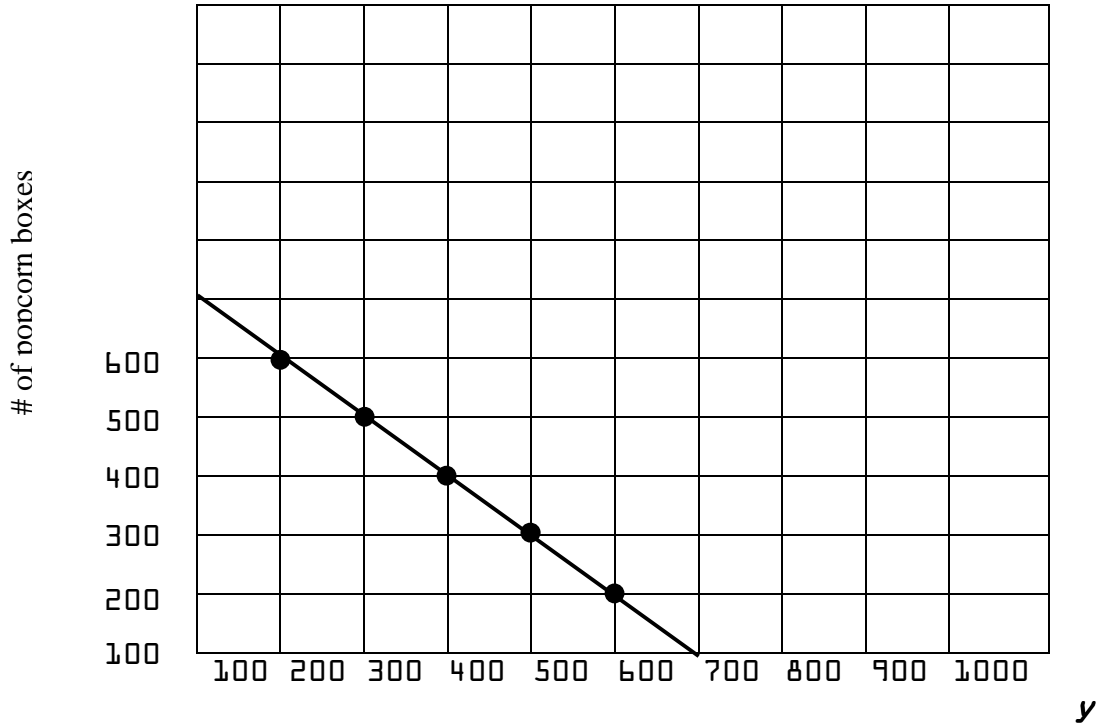
- A. Fruit = 150. They will need to sell 250 boxes of popcorn.
- B. See the grid on the next page.
- C. In order to find out how many boxes of popcorn they must sell you would refer to the graph. You would look on the x-axis for 600 which represents the 600 fruit bars. Then follow up that line until you reach the point. Then trace that back to the y-axis which represents the popcorn. It will read 200. You now know that you will need 200 popcorn boxes sold for a total of 120 and when added to the 600 fruit bars total of 180 you get 300 dollars.

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Score Point #4B continued

B.

x Fruit Bars & Popcorn sales (in hundreds)



Score Point 3

a) There is an expression you can use to solve this:

$$.60p + .80f = \quad p = \text{popcorn} \ \& \ f = \text{fruit}$$

$$.60p + .30(500) = 300$$

$$.60p - 150$$

$$p = 250$$

a:

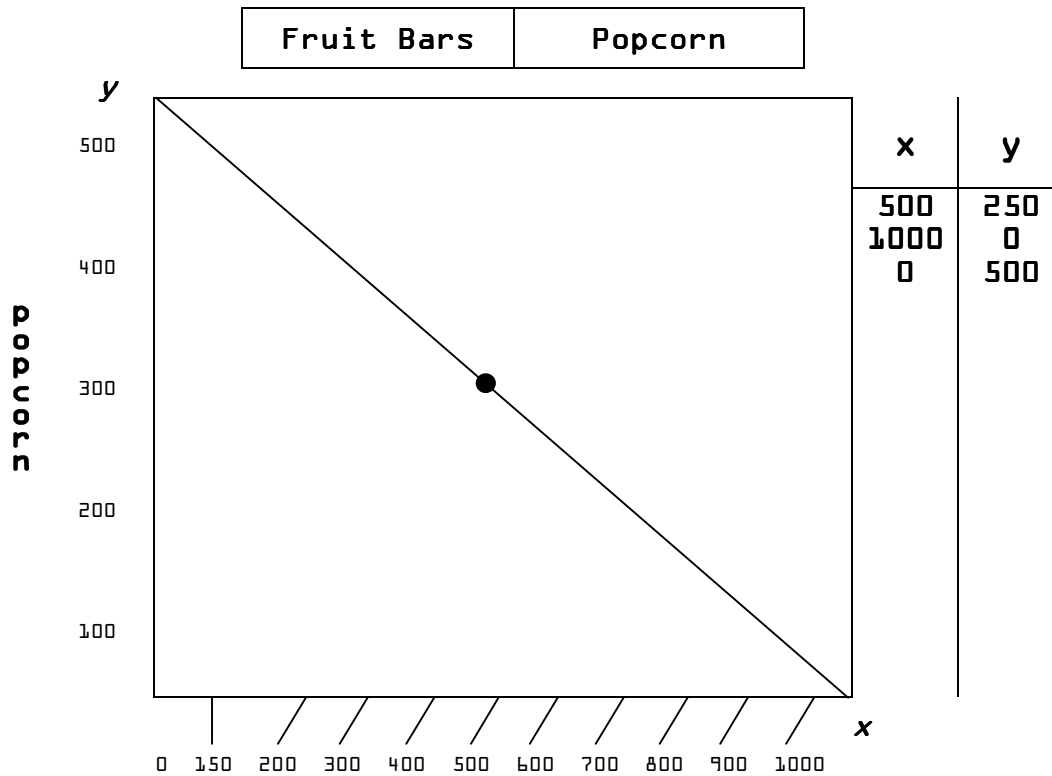
*they will need to sell 250 boxes of popcorn to have #300

b) See the graph on the next page.

c) My graph says that the students would have to sell 200 boxes of popcorn if they sell exactly 600 fruit bars to make a total of \$300.

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Score Point #3B continued



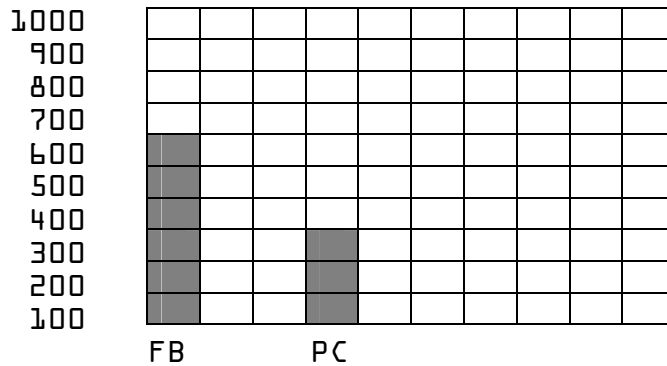
I used the equation $.60p + .30f = 300$

Score Point 2

- (A). If they sold 500 Fruit Bars at 0.30 cents they would have made \$150.00 dollars. I got this answer by multiplying 500 fruit bars by 0.30 cents and I got 250. So they would have to sell 250 bars of popcorn to make the goal of 300.00 dollars.
- (C). If the students sold 600 Fruit bars they would make \$180.00 dollars. The students would have to sell exactly 200 popcorn bags, to make 300.00 dollars.

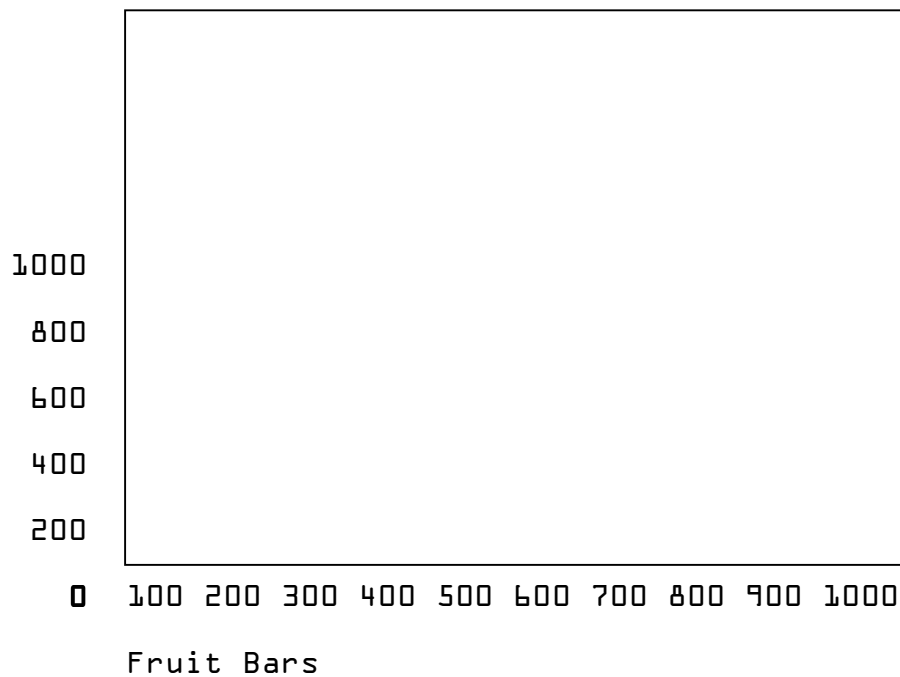
Mathematics, Grade 8

Score Point #2 continued



Score Point 1

- A. They will need 250 boxes of Popcorn.
- B. I will need twice as many popcorn as fruitbars.
- C. I would half to put the right amount of money on the graph.



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Score Point 0

A. / They would have to sell 18 boxes of popcorn.

B. /

60¢ popcorn	Fruit bars 30¢
5	
10	
100	
500	
800	
1000	

C. / 18 boxes of popcorn and 600 fruit bars.

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Comprehensive Assessment
System**

(MCAS)

Release of

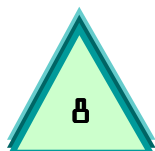
2000

Test Items

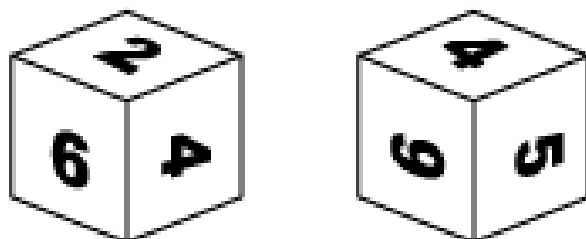
Mathematics

Grade 8

Session 1, Open Response Question #8



John is playing a board game that uses a pair of number cubes with sides numbered 1 to 6.



To find how many spaces he can move on the board, he adds the two numbers he rolls. The possible sums are:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

- Are all the sums John can roll equally likely? Explain your reasoning in detail.
- John needs to roll a sum of exactly 11 in order to get another turn. What is the probability that he will roll a sum of exactly 11? Explain your reasoning in detail.

Reporting Category Substrand for item 8: **Statistics and Probability; Probability.**

Question 8 Scoring Guide

Score	Description
4	The response demonstrates comprehensive understanding of the concepts of theoretical probability by accurately describing outcomes and events and determining the likelihood and probability of those events.
3	The response demonstrates general understanding of the concepts of theoretical probability by describing outcomes and events and their probability. Any errors are minor.
2	The response demonstrates basic understanding of some concepts of theoretical probability.
1	The response shows minimal understanding of some concepts of theoretical probability.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

- a. All the sums that John can roll out are not equally likely to happen. This occurs because some sums have more numbers than others that could be added to receive that sum. Here is an example:

1+1=2	2+1=3	3+1=4	4+1=5	5+1=6	6+1=7
1+2=3	2+2=4	3+2=5	4+2=6	5+2=7	6+2=8
1+3=4	2+3=5	3+3=6	4+3=7	5+3=8	6+3=9
1+4=5	2+4=6	3+4=7	4+4=8	5+4=9	6+4=10
1+5=6	2+5=7	3+5=8	4+5=9	5+5=10	6+5=11

As you can see the 7 occurs all 6 times while other numbers occur less. 2 occurs only once 3 occurs twice 4 occurs three times 8 occurs five times and so on

- b. The probability that John will roll an eleven is $\frac{1}{18}$. I found this answer by first finding two dice. It was $6 \times 6 = 36$. Now there were two ways to get an eleven. A 5 on the first dice and 6 on second dice or 6 on first dice and 5 on second dice. You get $\frac{2}{36}$ or $\frac{1}{18}$.

Score Point 3

- A. Not all of the sums John can roll are equally likely. Seven is the most likely to get because there are six different possibilities of rolling a seven.
- B. John has two chances of rolling an eleven he rolls a five or a five and a six. (See next page for details.)

Mathematics, Grade 8

Score Point #4B continued

2	1+1
3	1+2, 2+1
4	1+3, 2+2, 3+1
5	1+4, 2+3, 3+2, 4+1
6	1+5, 2+4, 3+3, 4+2, 5+1
7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1
8	2+6, 3+5, 4+4, 5+3, 6+2
9	3+6, 4+5, 5+4, 6+3
10	4+6, 5+5, 6+4
11	5+6, 6+5
12	6+6

Score Point 2

- a. Yes because there is equal probability.
- b. $\frac{1}{18}$ because there are 36 sums and 2 equal 11.

Score Point 1

- A. his sums are all equal because if each number cube adds up to six then you cant get higher than a 12.
- B. the probability that he will roll an 11 is 1 out of 36 because you can only get that number with a 6 and a 5 and there are 36 combinations with 2 number cubes so, there is a 1 out of 36of chance..

Score Point 0




- a. yes all sums ar equally likely to be rolled. This is because no matter what two numbers he rolls he will get one of these sums.
- b. His protability to roll an 11 is 1 out of 6. The only two numbers, he can get, to get 11 are 5 and 6 out of a total of 6 different numbers.




Session 1, Open Response Question #12 



12




Erin is writing a science fiction story. She has invented a money system for her planet that uses four coins that she drew and named like this:





CLUE 1:
3 s are worth the same as 1  and 1 .











CLUE 2:
Kay bought a game costing   .

She gave the clerk 2 s. Her change was 1 .

CLUE 3:
Kay and Max have the same amount of money.
Kay has 1 , 4 s, and 1 .

Max has 3 s and 6 s.

She has challenged her classmates to determine the relationships among the values of the coins from the following clues.

- Use Clue 1 above to find how many s equal 1 .
- Use Clue 2 to find out how many s equal 1 .
- Use Clue 3 and your answers to parts a and b to find how many s equal 1 . Use words or pictures to explain your reasoning.
- Erin told her classmates that 1  is worth 25¢ in U.S. money. What is the value in U.S. money of each of the following?
 - 1 
 - 1 
 - 1 

Reporting Category Substrand for item 12: *Patterns, Relations, and Functions/Algebra.*

Question 12 Scoring Guide

Score	Description
4	The response demonstrates thorough understanding of models of simple linear situations by correctly solving equations represented by models, clearly explaining the process, and evaluating the expressions for given values.
3	The response demonstrates general understanding of models of simple linear situations by solving equations represented by models, explaining the process, and evaluating the expressions for given values with only minor errors.
2	The response shows basic understanding of the concept of simple linear situations by solving some equations and/or evaluating some expressions for given values.
1	The response shows minimal understanding of simple linear situations.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

a. $2\textcircled{C}s = 1\textcircled{A}$. I started with $3\textcircled{C} = 1\textcircled{A} + 1\textcircled{C}$, then subtracted $1\textcircled{C}$ from end equation.

b. $3\textcircled{R}s = 1\textcircled{H}$. I set up the equation as $\textcircled{H} + 2\textcircled{R} = 2\textcircled{H} - 1\textcircled{R}$. First, I added \textcircled{R} to site of the equation, leaving $\textcircled{H} + 3\textcircled{R} = 2\textcircled{H}$. Then I subtracted $1\textcircled{H}$ from each side to get $3\textcircled{R}$.

Mathematics, Grade 8

Score Point #4C continued

c. $4\triangle = 1\text{R}$. I set up the equation as $1\text{R} + 4\text{C} + 1\text{H} = 3\text{R} + 6\triangle$. First, I reconciled the 3R because I knew from part b. that they are equal, leaving $1\text{R} + 4\text{C} + 6\triangle$. I knew from $2\text{C} = 1\triangle$ and therefore that $*\text{C} = 2\triangle$, so I subtracted the 4C and $2\triangle$ leaving $1\triangle = 4$.

d. $1\triangle = 50\text{¢}$
 $1\text{R} = \$2.00$
 $1\text{H} = \$6.00$

Score Point 3

a. 3C 's are equal to $1\triangle$. This is stated in clue 1.

$$2\text{C}'s = 1\text{T} \quad \cancel{4\text{C}} = \cancel{1\text{T}} \quad \text{C}\text{C} = \triangle$$

b. I did guess and check with $\text{H}=9$ and $\text{R}=3$. This worked.

$$\begin{array}{l} \text{H}=9 \quad 9 \cdot 3 = 27 \\ \text{R}=3 \quad 3 \cdot 15 = 45 \\ \text{H} > \text{R} \end{array} \quad \text{H} = 3\text{R}'s$$

c. $1\text{R} = 3\text{T}$ $2 = \text{T}$ $\text{R} > \text{T}$
 RCH $1\text{H} = 3\text{R}$
 3RT

$$\begin{array}{l} 1\text{R} + 4\text{C} + 1\text{T} = 3\text{R} + 6\triangle \\ 1\text{R} + 2\text{C} + 6\text{T} \\ 1\text{R} + 1\text{C} + 5\text{T} \end{array}$$

$$2\text{C} - 1\text{T} \quad 1\text{T} = 3\text{R} \quad 1\text{R} = 3\text{T}$$

d. $\text{C} = .25$ 25
 $\text{T} = .50$ $\underline{\quad 2}$
 $\text{R} = \$1.50$ 50
 $\text{H} = \$4.50$ $.50$

$$\begin{array}{r} \underline{\quad 3} \\ \$1.50 \\ \underline{\quad 150} \\ \quad \quad 3 \\ \underline{\quad \quad 3} \\ \$4.50 \end{array}$$

Score Point 2

- A. $\textcircled{C}\textcircled{C}\textcircled{C} = \triangle & \textcircled{C}$
 $\textcircled{C}\textcircled{C}\textcircled{C}\textcircled{C}\textcircled{C}\textcircled{C} = \triangle\triangle & \textcircled{C}\textcircled{C}$ -- two \textcircled{C} are in \triangle
 You can automatically cross out two \textcircled{C} 's from the equation because they have the same value.
 $\textcircled{C}\textcircled{C}\textcircled{C}\textcircled{C} = \triangle\triangle$ } If two $\triangle =$ four \textcircled{C} 's
 then one $\triangle =$ two \textcircled{C} 's
- B. \square 's are less than \hexagon
 Game costs $\hexagon\square\square$
 Paid with two \hexagon 's
 Three \square 's in an \hexagon
- C.
- D. $\textcircled{C} = 25\text{¢}$
 $\triangle = 50\text{¢}$
 $\square = 75\text{¢}$
 $\hexagon = \$2.25$

Score Point 1

- a. 2 \textcircled{C} 's are equal to 1 \triangle
 b. 2 \square 's = 1 \hexagon
 c.

Score Point 0

- a) $1\frac{1}{2}\textcircled{C}$'s are equal to 1 \triangle
 b) a $\frac{1}{2}\square$ equals to 1 \hexagon
 c) 2 \triangle 's equal 1 \square
 d) 1 $\triangle = 12\frac{1}{2}\text{¢}$ 1 $\square = 50\text{¢}$ 1 $\hexagon = 10\text{¢}$

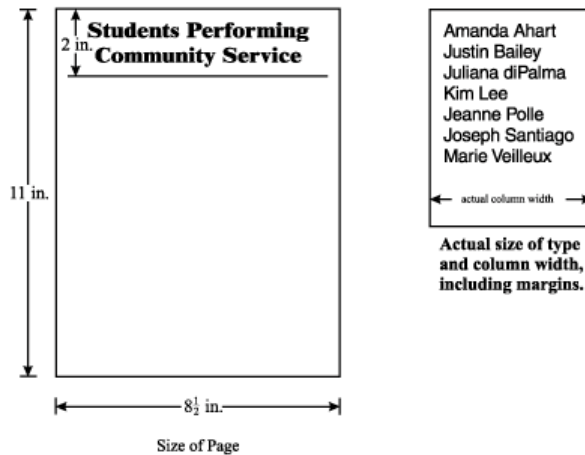
Session 2, Open Response Question #23

Use your ruler to answer this open-response question.



Jarrold is the editor of the school newspaper. In the next issue, a page will be devoted to a list of the students who perform community service. Jarrold is planning how to arrange the names.

The first figure below tells the size of the page and the headline. The second figure shows the actual size of type that will be used for the students' names and the actual width of each column.



There are 175 students who performed community service. Jarrold wants to plan the page so that

- the page has the greatest number of columns possible, and
 - the columns are as close to the same length as possible.
- a. What is the greatest number of columns that Jarrold can put on the page? Show or describe how you found your answer.
 - b. How many names should he put in each column so that the columns are of equal length or as close to equal length as possible? Assume each name will fit on one line in a column.
 - c. How long will each column of names be? Show or describe how you found your answer.

Reporting Category Substrand for item 23: **Geometry and Measurement/Measurement.**

Mathematics, Grade 8

Question 23 Scoring Guide

Score	Description
4	The response demonstrates comprehensive understanding of the concept of measurement by solving a real-life problem involving the selection of an appropriate measurement tool and a correct strategy for making the measurement.
3	The response demonstrates a practical understanding of the concept of measurement by solving a real-life problem involving the selection of an appropriate measurement tool and a correct strategy for making the measurement with only one or two computational errors.
2	The response shows partial understanding of the concept of measurement by attempting to solve a real-life problem by selecting an appropriate measurement tool but not using a correct strategy for making the measurement.
1	The response shows minimal understanding of the concept of measurement.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Score Point 4

- a. The greatest number of columns Jarrod can put on the page is 5. I figured this out by measuring the column width: $1\frac{1}{2}$ in, and the page's width: $8\frac{1}{2}$ in. I then divided 1.5 into 8.5 : 5.6 and since you can't have half a column, I rounded it down to 5.
- b. He should put 35 names in each column. To figure that out, I simply divided 5 (#of columns) into 175 (#of names).
- c. The columns will be 6.5 inches long. I figured that out by measuring how long the 7 names they gave us together were and they were about 1.3 inches long. There were 7 names and 7 goes into 35 5 times, so I then just multiplied $1 \times 3 \times 5$ and got 6.5.

Score Point 3

- A. The greatest number of columns would be $8\frac{1}{2} \div 1\frac{1}{2}$ (The smallest column size). So, Jarrod can fit 5 because 1 inch is left over. this can be used for a half inch border.
- B. He should put 35 names in each row because $175 \text{ names} \div 5 \text{ columns} = 35 \text{ names} \frac{\text{names}}{\text{column}}$.
- C. Well, Since each text name is $\frac{1}{8}$ tall and there are $\frac{1}{16}$ inch squares between names with a $\frac{1}{8}$ inch top and bottom border.

$35 \text{ names} * \frac{1}{8} \text{ inch} = 4 \text{ inches } 2\frac{7}{8}$	$= \frac{39}{8}$
$1 \text{ top} \& \ 1 \text{ bottom border} = \frac{2}{8} \text{ inch}$	$\frac{2}{8}$
$34 \text{ spaces} * \frac{1}{16} = 2\frac{1}{8} \text{ inch}$	$+ \frac{17}{8}$
Total Column	$\frac{58}{8} = 7.25$ or

Each column is $7\frac{1}{4}$ inches long.

Score Point 2

- a) The greatest amount of columns that Jarrod can put on his page is five, because if each column is $1\frac{1}{2}$ inches wide, only five columns can fit into his $8\frac{1}{2}$ inches wide page. This was figured out by doing 1.5×5 which is 7.5.
- b) There should be 35 names per column, because 5 goes into 175 evenly.
- c) Each column will be 9 inches long, because if his heading takes up two inches at the top of the page, a 9 inch column would fit the rest of the page.

Score Point 1

6 names = $1\frac{1}{2}$ " width
 $1\frac{1}{2}$ " long

a. $8\frac{1}{2} \div 1\frac{1}{2} = 5\frac{2}{3}$ columns

b. $6 \times 6 = 36$ names in each column

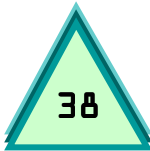
c. 11 inches - 2 inches = 9 inches

you need to subtract 2" for the heading.

Score Point 0

- a. The greatest number of columns that you could put on this page is 2 columns.
- b. I think that he can put 87 on one side and 88 on the other. Because if you do $8\frac{1}{2} \times 11$ you get $89\frac{1}{2}$ but since you sould to be as accurate as possible so I would put 88 on one side and 87 on the other.
- c. I think each column will be five inches because it was eleven but you need to subtract the two inches for the heading and you get nine.

Session 3, Open Response Question #38



The planning committee at Lane Middle School is planning a pizza party for its 127 eighth-grade students. They got this menu from The Pizza Palace. The planning committee took a survey of a random sample of 26 eighth-grade students by asking, "What kind of pizza do you want?" This is what they found.

The Pizza Palace		
FREE DELIVERY		
PIZZA IS OUR SPECIALTY		
	Medium (Serves 4)	Large (Serves 6)
Cheese.....	\$9.00	\$11.00
Sausage.....	\$9.75	\$12.00
Pepperoni.....	\$9.75	\$12.00
Vegetarian.....	\$9.50	\$11.75

Favorite Kind of Pizza				
Kind of Pizza	Cheese	Sausage	Pepperoni	Vegetarian
Number of students	7	3	9	7

The committee has a budget of \$300 for the pizza. What kinds and sizes of pizzas could the committee order so that each of the 127 students can have his or her favorite kind of pizza?

- Explain how you used the results of the survey to decide which pizzas to order.
- Show or describe the calculations needed to be sure that there will be enough pizza for the 127 students.
- Show or describe the calculations needed to be sure that the cost of the pizza totals \$300 or less.

You do not need to find the cheapest way to buy enough pizza. You only need to make sure that the total cost is \$300 or less.

Reporting Category Substrand for item 38: **Number Sense/Computation and Estimation.**

Question 38 Scoring Guide

Score	Description
4	The response demonstrates comprehensive understanding of data analysis by accurately displaying data and by correctly recognizing, extending, and describing the patterns displayed to solve a real-life problem.
3	The response demonstrates general understanding of data analysis by displaying data and by recognizing, extending, and describing the patterns displayed to solve a real-life problem. Any errors are minor.
2	The response demonstrates a basic understanding of data analysis by displaying data or by recognizing, extending, and/or describing the patterns displayed to solve a real-life problem.
1	The response shows minimal understanding of some concepts of data analysis.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

Mathematics, Grade 8

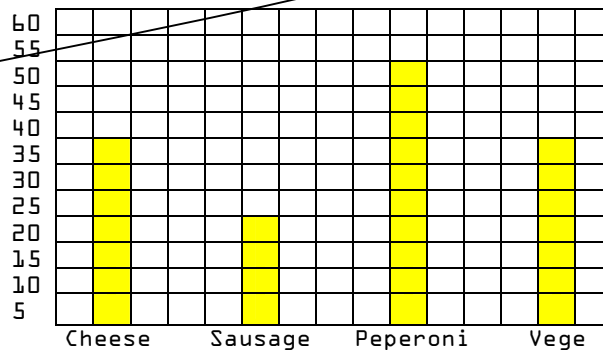
Score Point 4

I first divided the number of students that were surveyed into the total number of students. I set 5 for each 1 student surveyed then I multiplied how many students liked each kind of pizza by 5.

$$\begin{array}{r}
 488 \\
 26 \overline{)12700} \\
 \underline{52} \\
 15 \\
 \underline{15} \\
 00
 \end{array}$$

Sausage 15	cost \$
2 L's	24.00
1 med cheese 35	9.75
6L peperonie	66.00
7L 1m veggie 35	84.00
	9.75
6L	70.50
total	264.00\$

Then I decided how many of each pizza I should get and calculated the prices. The total price was \$264.00 so you should follow my chart shown here.



Score Point 3

- A. The way I used the results was by taking each vote and turning it into what the vote each number would be if 11 were R7 people. I multiplied that by 12? From there I figured out how many large pizzas the I would need for each value. I did this by dividing each number by six. Then I added up all the prices and got \$264.

Mathematics, Grade 8

Score Point #3 continued

B. First I:

sausage=3 pepperoni=9 cheese=7 vegetarian=7

$$\begin{array}{r} .+7 \\ 26 \overline{) 3} \end{array}$$
 = always rounded up so there would be enough pizza.

then I:

$127 \times .12 = 15.24 = 15$
 people for sausage rounded down

pepperoni

$$\begin{array}{r} .35 \\ 26 \overline{) 9} \end{array}$$
 then

$127 \times .35 = 44.45 = 44$
 people for pepperoni

So then
 x5 x6 move all the numbers and divide by 6 and round up so there will be more than enough for everyone. By doing so, the amount is very close.

Cheese:
$$\begin{array}{r} .27 \\ 26 \overline{) 7} \end{array}$$
 $127 \times 27 = 54$
 and 39 people for cheese
 39 people for vegetarian

C.

$\begin{array}{r} 6 \\ 6 \overline{) 34} \end{array}$	$\begin{array}{r} 6 \\ 6 \overline{) 34} \end{array}$	$\begin{array}{r} 8 \\ 6 \overline{) 44} \end{array}$	$\begin{array}{r} 5 \\ 6 \overline{) 15} \end{array}$
-------------------------------------------------------	-------------------------------------------------------	-------------------------------------------------------	-------------------------------------------------------

Round up \blacktriangle so there's enough

$6 \times 12.00 = \$55$	
$6 \times 11.75 = \$70.5$	
$8 \times 12.00 = \$96$	
$3 \times 12.00 = \underline{\$36}$	With Change
$\$268.50$	to spare!

Score Point 2

I would recommend that you should buy
6 large cheese-pizzas, coming to \$66.00
7 large pepperoni pizzas, coming to \$84.00
5 large vegetarian pizzas for \$48.00
The large pizzas serve 6 kids. There are 127.
If you divide $127 \div 6 = 21.17$ You have a total
of 22 pizzas, which I recommend 22 pizzas is
enough for every kid and the total price of the
pizzas are \$256.75. Which is less than the
\$300 budget.

Score Point 1

Vegetarian - should order a bout 10 smalls
Peperoni - should order about 4 large
Sausage - should order about 4 small
Cheese - order 4 large 2 small
95.00
48.00
39.00
62.00
\$244.00

Score Point 0

There would be 10 cheese pizzas 5 sausage pizzas
15 pepperoni pizzas and 10 vegetarian pizzas.

Session 3, Open Response Question #39



For Tiffany and Miguel's science fair project, they dropped the same ball from a height of 200 centimeters 20 times. Each time they dropped the ball, they measured how high it bounced on its first four bounces. The table below gives the average of their measurements.

Height of 1st bounce	153 cm
Height of 2nd bounce	110 cm
Height of 3rd bounce	86 cm
Height of 4th bounce	63 cm

- Using the grid in your answer booklet, draw a graph showing the data in the table. Be sure to label the axes.
- Predict the height of the 5th bounce.
- Describe the pattern that can be used to predict the height of the bounce.

Reporting Category Substrand for item 39: **Statistics and Probability/Statistics.**

Mathematics, Grade 8

Question 39 Scoring Guide

Score	Description
4	The response demonstrates comprehensive understanding of the concept of ratios by using correct calculations and inferences based on data and constraints to solve real-life problems and then explaining how the data were used.
3	The response demonstrates general understanding of the concepts of ratios by using appropriate calculations and inferences based on data and constraints to solve real-life problems and then explaining how the data were used. Any errors are minor and explanations may be vague.
2	The response demonstrates a basic understanding of some concepts of ratios but does not consider all of the constraints.
1	The response shows minimal understanding of ratios.
0	The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.
Blank	No response.

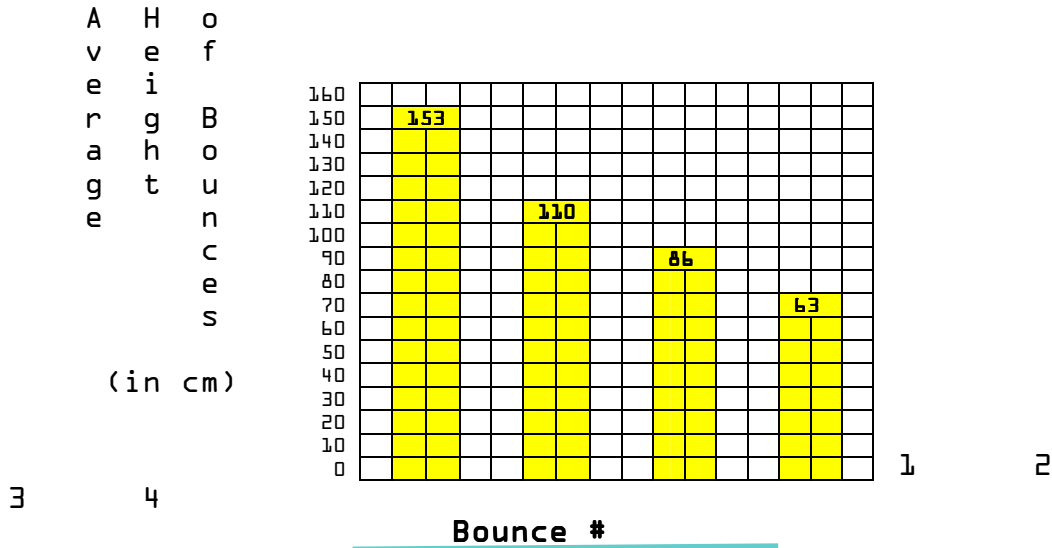
Score Point 4

- B. I predict that on the 5th bounce the ball will go 47cm high.
- C. It was rather complicated estimating how estimating how high the ball would go on the 5th bounce, but I used a pattern from the first four bounces. I figured out that the second bounce was 71% as high as the first, the third was 78% as high as the second, and the fourth was 73% as high as the third. To figure out what percentage the fifth bounce would be of the fourth, I averaged these percents together and got 74%. Then 74% of the fourth bounce, which was 63cm, would be the height of the fifth. 74% of 63 is 47 and that is where I got my prediction. (See next page for graph.)

Mathematics, Grade 8

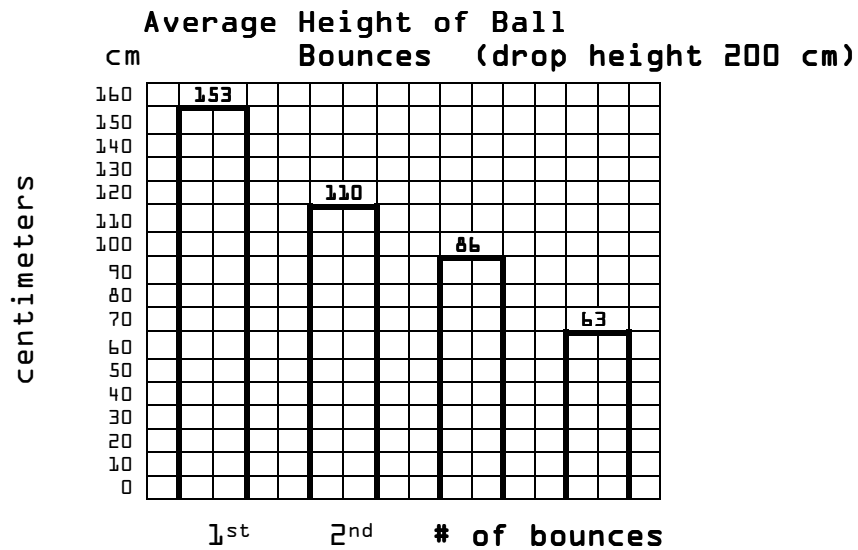
Score Point #4 continued

A. Ball Bounces



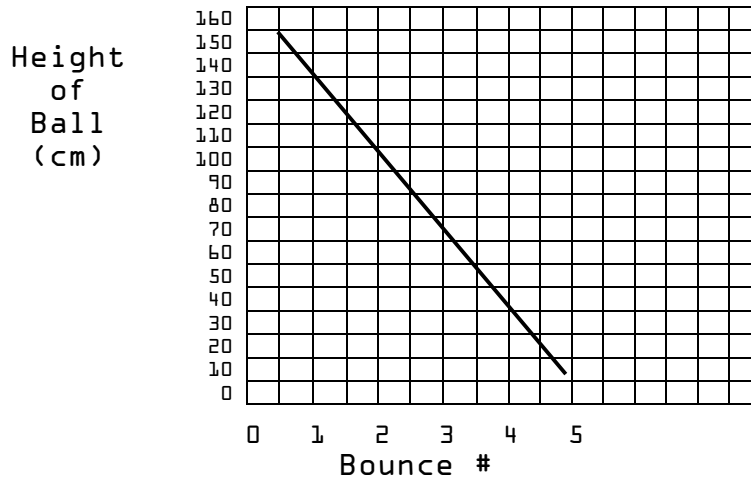
Score Point 3

- B. My prediction for the 5th bounce is that the ball will bounce back to 51cm
- C. The pattern for each bounce is that each time it bounces it bounces about 20% less higher than it did the last time.



Score Point 2

I predict that the fifth bounce will be at 46cm, because that is the pattern it has been going so far. After the first bounce there was a difference of 43, but then the differences went down to 34 & 23. You can predict the bounces by assuming it will bounce about 24cm less each time.



Score Point 1

Height of Drop	200cm		
Height of 1 st bounce	153cm	43	47
Height of 2 nd bounce	110cm	24	90
Height of 3 rd bounce	86cm	23	114
Height of 4 th bounce	63cm		137



- A.
- B. the 5th bounce will be 43cm
- C. you subtract 200 and the next number that bounces.

Score Point 0

A.

Average Height	
1 st Bounce	153cm
2 nd Bounce	110cm
3 rd	86cm
4 th	63cm

B.

73cm

C.

count the Height